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MAINTAINING A DOUBLE-ENDED QUEUE AS A LINKED-LIST WITH SENTINEL NODES AND DELETE FLAGS WITH CONCURRENT NON-BLOCKING INSERT AND REMOVE OPERATIONS USING A DOUBLE COMPARE-AND-SWAP PRIMITIVE

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[1001] This application claims benefit of U.S. Provisional Application No. 60/177,090, filed January 20, 2000, which is incorporated in its entirety herein by reference.

BACKGROUND OF THE INVENTION

Field of the Invention

[1002] The present invention relates to coordination amongst processors in a multiprocessor computer, and more particularly, to structures and techniques for facilitating non-blocking access to concurrent shared objects.

Description of the Related Art

[1003] Non-blocking algorithms can deliver significant performance benefits to parallel systems. However, there is a growing realization that existing synchronization operations on single memory locations, such as compare-and-swap (CAS), are not expressive enough to support design of efficient non-blocking algorithms. As a result, stronger synchronization operations are often desired. One candidate among such operations is a double-word compare-and-swap (DCAS). If DCAS operations become more generally supported in computers systems and, in some implementations, in hardware, a collection of efficient current data structure implementations based on the DCAS operation will be needed.

[1004] Massalin and Pu disclose a collection of DCAS-based concurrent algorithms. See e.g., H. Massalin and C. Pu, *A Lock-Free Multiprocessor OS Kernel*, Technical

Report TR CUCS-005-9, Columbia University, New York, NY, 1991, pages 1-19. In particular, Massalin and Pu disclose a lock-free operating system kernel based on the DCAS operation offered by the Motorola 68040 processor, implementing structures such as stacks, FIFO-queues, and linked lists. Unfortunately, the disclosed algorithms are centralized in nature. In particular, the DCAS is used to control a memory location common to all operations, and therefore limits overall concurrency.

[1005] Greenwald discloses a collection of DCAS-based concurrent data structures that improve on those of Massalin and Pu. *See e.g.*, M. Greenwald, *Non-Blocking Synchronization and System Design*, Ph.D. thesis, Stanford University Technical Report STAN-CS-TR-99-1624, Palo Alto, CA, 8 1999, 241 pages. In particular, Greenwald discloses implementations of the DCAS operation in software and hardware and discloses two DCAS-based concurrent double-ended queue (deque) algorithms implemented using an array. Unfortunately, Greenwald's algorithms use DCAS in a restrictive way. The first, described in Greenwald, *Non-Blocking Synchronization and System Design*, at pages 196-197, used a two-word DCAS as if it were a three-word operation, storing two deque end pointers in the same memory word, and performing the DCAS operation on the two pointer word and a second word containing a value. Apart from the fact that Greenwald's algorithm limits applicability by cutting the index range to half a memory word, it also prevents concurrent access to the two ends of the deque. Greenwald's second algorithm, described in Greenwald, *Non-Blocking Synchronization and System Design*, at pages 217-220) assumes an array of unbounded size, and does not deal with classical array-based issues such as detection of when the deque is empty or full.

[1006] Arora et al. disclose a CAS-based deque with applications in job-stealing algorithms. *See e.g.*, N. S. Arora, Blumofe, and C. G. Plaxton, *Thread Scheduling For Multiprogrammed Multiprocessors*, in *Proceedings of the 10th Annual ACM Symposium on Parallel Algorithms and Architectures*, 1998. Unfortunately, the disclosed non-blocking implementation restricts one end of the deque to access by only a single processor and restricts the other end to only pop operations.

[1007] Accordingly, improved techniques are desired that do not suffer from the above-described drawbacks of prior approaches.

SUMMARY

[1008] A set of structures and techniques are described herein whereby an exemplary concurrent shared object, namely a double-ended queue (deque), is provided.

Although a described non-blocking, linearizable deque implementation exemplifies several advantages of realizations in accordance with the present invention, the present invention is not limited thereto. Indeed, based on the description herein and the claims that follow, persons of ordinary skill in the art will appreciate a variety of concurrent shared object implementations. For example, although the described deque implementation exemplifies support for concurrent push and pop operations at both ends thereof, other concurrent shared objects implementations in which concurrency requirements are less severe, such as LIFO or stack structures and FIFO or queue structures, may also be implemented using the techniques described herein.

[1009] Accordingly, a novel linked-list-based concurrent shared object implementation has been developed that provides non-blocking and linearizable access to the concurrent shared object. In an application of the underlying techniques to a deque, the linked-list-based algorithm allows non-blocking completion of access operations without restricting concurrency in accessing the deque's two ends. The new implementation is based at least in part on a new technique for splitting a pop operation into two steps, marking that a node is about to be deleted, and then deleting it. Once marked, the node is logically deleted, and the actual deletion from the list can be deferred. In one realization, actual deletion is performed as part of a next push or pop operation performed at the corresponding end of the deque. An important aspect of the overall technique is synchronization of delete operations when processors detect that there are only marked nodes in the list and attempt to delete one or more of these nodes concurrently from both ends of the deque.

[1010] A novel array-based concurrent shared object implementation has also been developed, which provides non-blocking and linearizable access to the concurrent shared object. In an application of the underlying techniques to a deque, the array-based algorithm allows uninterrupted concurrent access to both ends of the deque, while returning appropriate exceptions in the boundary cases when the deque is empty or full. An interesting characteristic of the concurrent deque implementation is that a

processor can detect these boundary cases, e.g., determine whether the array is empty or full, without checking the relative locations of the two end pointers in an atomic operation.

[1011] Both the linked-list-based implementation and the array-based implementation provide a powerful concurrent shared object construct that, in realizations in accordance with the present invention, provide push and pop operations at both ends of a deque, wherein each execution of a push or pop operation is non-blocking with respect to any other. Significantly, this non-blocking feature is exhibited throughout a complete range of allowable deque states. For an array-based implementation, the range of allowable deque states includes full and empty states. For a linked-list-based implementation, the range of allowable deque states includes at least the empty state, although some implementations may support treatment of a generalized out-of-memory condition as a full state.

BRIEF DESCRIPTION OF THE DRAWINGS

[1012] The present invention may be better understood, and its numerous objects, features, and advantages made apparent to those skilled in the art by referencing the accompanying drawings.

[1013] **FIGS. 1A and 1B** illustrate exemplary empty and full states of a double-ended queue (deque) implemented as an array in accordance with the present invention.

[1014] **FIG. 2** illustrates successful operation of a `pop_right` operation on a partially full state of a deque implemented as an array in accordance with the present invention.

[1015] **FIG. 3** illustrates successful operation of a `push_right` operation on a empty state of a deque implemented as an array in accordance with the present invention.

[1016] **FIG. 4** illustrates contention between opposing `pop_left` and `pop_right` operations for a single remaining element in an almost empty state of a deque implemented as an array in accordance with the present invention.

[1017] **FIGS. 5A, 5B and 5C** illustrate the results of a sequence of `push_left` and `push_right` operations on a nearly full state of a deque implemented as an array in accordance with the present invention. Following successful completion of the `push_right` operation, the deque is in a full state. **FIGS. 5A, 5B and 5C** also illustrate an artifact of the linear depiction of a circular buffer, namely that, through a series of preceding operations, ends of the deque may wrap around such that left and right indices may appear (in the linear depiction) to the right and left of each other.

[1018] **FIG. 6** depicts an alternative deleted node indication encoding technique employing a dummy node suitable for use in a linked-list-based implementation of a deque.

[1019] **FIGS. 7A, 7B, 7C and 7D** depict various empty states of a deque implemented as a doubly linked-list in accordance with an exemplary embodiment of the present invention. **FIGS. 7B, 7C and 7D** depict valid empty states that may occur in a linked-list-based implementation of a deque after successful completion of a `pop_left` or `pop_right` operation, but before successful execution of an appropriate null node deletion operation.

[1020] **FIGS. 8A and 8C** depict valid deque states before and after successful completion of a `delete_right` operation in accordance with an exemplary doubly linked-list embodiment of the present invention. **FIGS. 8B and 8D** depict valid deque states before and after successful completion of a `pop_right` operation in accordance with an exemplary doubly linked-list embodiment of the present invention.

[1021] **FIGS. 9A and 9B** depict execution of a `push_right` access operation for a deque implemented as doubly linked-list in accordance with an exemplary embodiment of the present invention. In particular, **FIGS. 9A and 9B** illustrate a deque state before and after successful completion of a synchronization operation.

[1022] **FIG. 10** illustrates two valid outcomes in an execution sequences wherein competing concurrent `left_delete` and `right_delete` operations operate on a empty deque state with two null elements.

[1023] The use of the same reference symbols in different drawings indicates similar or identical items.

DESCRIPTION OF THE PREFERRED EMBODIMENT(S)

[1024] The description that follows presents a set of techniques, objects, functional sequences and data structures associated with concurrent shared object implementations employing double compare-and-swap (DCAS) operations in accordance with an exemplary embodiment of the present invention. An exemplary non-blocking, linearizable concurrent double-ended queue (deque) implementation is illustrative. A deque is a good exemplary concurrent shared object implementation, in that it involves all the intricacies of LIFO-stacks and FIFO-queues, with the added complexity of handling operations originating at both of the deque's ends.

Accordingly, techniques, objects, functional sequences and data structures presented in the context of a concurrent deque implementation will be understood by persons of ordinary skill in the art to describe a superset of support and functionality suitable for less challenging concurrent shared object implementations, such as LIFO-stacks, FIFO-queues or concurrent shared objects (including deques) with simplified access semantics.

[1025] In view of the above, and without limitation, the description that follows focuses on an exemplary linearizable, non-blocking concurrent deque implementation which behaves as if access operations on the deque are executed in a mutually exclusive manner, despite the absence of a mutual exclusion mechanism.

Advantageously, and unlike prior approaches, deque implementations in accordance with some embodiments of the present invention allow concurrent operations on the two ends of the deque to proceed independently.

Computational Model

[1026] One realization of the present invention is as a deque implementation, employing the DCAS operation, on a shared memory multiprocessor computer. This realization, as well as others, will be understood in the context of the following computation model, which specifies the concurrent semantics of the deque data structure.

[1027] In general, a *concurrent system* consists of a collection of n *processors*. Processors communicate through shared data structures called *objects*. Each object has an associated set of primitive *operations* that provide the mechanism for manipulating that object. Each processor P can be viewed in an abstract sense as a sequential thread of control that applies a sequence of operations to objects by issuing an invocation and receiving the associated response. A *history* is a sequence of invocations and responses of some system execution. Each history induces a “real-time” order of operations where an operation A *precedes* another operation B , if A ’s response occurs before B ’s invocation. Two operations are *concurrent* if they are unrelated by the real-time order. A *sequential history* is a history in which each invocation is followed immediately by its corresponding response. The *sequential specification* of an object is the set of *legal* sequential histories associated with it. The basic correctness requirement for a concurrent implementation is *linearizability*. Every concurrent history is “equivalent” to some legal sequential history which is consistent with the real-time order induced by the concurrent history. In a linearizable implementation, an operation appears to take effect atomically at some point between its invocation and response. In the model described herein, a shared memory location L of a multiprocessor computer’s memory is a linearizable implementation of an object that provides each processor P_i with the following set of sequentially specified machine operations:

$Read_i(L)$ reads location L and returns its value.

$Write_i(L, v)$ writes the value v to location L .

$DCAS_i(L1, L2, o1, o2, n1, n2)$ is a double compare-and-swap operation with the semantics described below.

[1028] Implementations described herein are *non-blocking* (also called *lock-free*). Let us use the term *higher-level operations* in referring to operations of the data type being implemented, and *lower-level operations* in referring to the (machine) operations in terms of which it is implemented. A non-blocking implementation is one in which even though individual higher-level operations may be delayed, the system as a whole continuously makes progress. More formally, a *non-blocking* implementation is one in which any history containing a higher-level operation that has an invocation but no response must also contain infinitely many responses

concurrent with that operation. In other words, if some processor performing a higher-level operation continuously takes steps and does not complete, it must be because some operations invoked by other processors are continuously completing their responses. This definition guarantees that the system as a whole makes progress and that individual processors cannot be blocked, only delayed by other processors continuously taking steps. Using locks would violate the above condition, hence the alternate name: *lock-free*.

Double-word Compare-and-Swap Operation

[1029] Double-word compare-and-swap (DCAS) operations are well known in the art and have been implemented in hardware, such as in the Motorola 68040 processor, as well as through software emulation. Accordingly, a variety of suitable implementations exist and the descriptive code that follows is meant to facilitate later description of concurrent shared object implementations in accordance with the present invention and not to limit the set of suitable DCAS implementations. For example, order of operations is merely illustrative and any implementation with substantially equivalent semantics is also suitable. Furthermore, although exemplary code that follows includes overloaded variants of the DCAS operation and facilitates efficient implementations of the later described push and pop operations, other implementations, including single variant implementations may also be suitable.

```
boolean DCAS(val *addr1, val *addr2,
             val old1, val old2,
             val new1, val new2) {
    atomically {
        if ((*addr1==old1) && (*addr2==old2)) {
            *addr1 = new1;
            *addr2 = new2;
            return true;
        } else {
            return false;
        }
    }
}
```



```

boolean DCAS(val *addr1, val *addr2,
             val old1, val old2,
             val *new1, val *new2) {
    atomically {
        temp1 = *addr1;
        temp2 = *addr2;
        if ((temp1 == old1) && (temp2 == old2)) {
            *addr1 = *new1;
            *addr2 = *new2;
            *new1 = temp1;
            *new2 = temp2;
            return true;
        } else {
            *new1 = temp1;
            *new2 = temp2;
            return false;
        }
    }
}

```

[1030] Note that in the exemplary code, the DCAS operation is overloaded, i.e., if the last two arguments of the DCAS operation (new1 and new2) are pointers, then the second execution sequence (above) is operative and the original contents of the tested locations are stored into the locations identified by the pointers. In this way, certain invocations of the DCAS operation may return more information than a success/failure flag.

[1031] The above sequences of operations implementing the DCAS operation are executed atomically using support suitable to the particular realization. For example, in various realizations, through hardware support (e.g., as implemented by the Motorola 68040 microprocessor or as described in M. Herlihy and J. Moss, *Transactional memory: Architectural Support For Lock-Free Data Structures*, Technical Report CRL 92/07, Digital Equipment Corporation, Cambridge Research Lab, 1992, 12 pages), through non-blocking software emulation (such as described in G. Barnes, *A Method For Implementing Lock-Free Shared Data Structures*, in *Proceedings of the 5th ACM Symposium on Parallel Algorithms and Architectures*, pages 261–270, June 1993 or in N. Shavit and D. Touitou, *Software transactional memory, Distributed Computing*, 10(2):99–116, February 1997), or via a blocking software emulation (such as described in U.S. Patent Application No. XX/xxx,xxx, entitled “PLATFORM INDEPENDENT DOUBLE COMPARE AND SWAP

OPERATION," naming Cartwright and Agesen as inventors, and filed December 9, 1998).

[1032] Although the above-referenced implementations are presently preferred, other DCAS implementations that substantially preserve the semantics of the descriptive code (above) are also suitable. Furthermore, although much of the description herein is focused on double-word compare-and-swap (DCAS) operations, it will be understood that N-location compare-and-swap operations ($N \geq 2$) may be more generally employed, though often at some increased overhead.

A Double-ended Queue (Deque)

[1033] A *deque* object S is a concurrent shared object, that in an exemplary realization is created by an operation of a constructor operation, e.g., `make_deque(length_s)`, and which allows each processor P_i , $0 \leq i \leq n-1$, of a concurrent system to perform the following types of operations on S : `push_righti(v)`, `push_lefti(v)`, `pop_righti()`, and `pop_lefti()`. Each push operation has an input, v , where v is selected from a range of values. Each pop operation returns an output from the range of values. Push operations on a full deque object and pop operations on an empty deque object return appropriate indications.

[1034] A concurrent implementation of a deque object is one that is linearizable to a standard sequential deque. This sequential deque can be specified using a state-machine representation that captures all of its allowable sequential histories. These sequential histories include all sequences of push and pop operations induced by the state machine representation, but do not include the actual states of the machine. In the following description, we abuse notation slightly for the sake of clarity.

[1035] The state of a deque is a sequence of items $S = \langle v_0, \dots, v_k \rangle$ from the range of values, having cardinality $0 \leq |S| \leq \text{length_s}$. The deque is initially in the empty state (following invocation of `make_deque(length_s)`), that is, has cardinality 0, and is said to have reached a full state if its cardinality is `length_s`.

[1036] The four possible push and pop operations, executed sequentially, induce the following state transitions of the sequence $S = \langle v_0, \dots, v_k \rangle$, with appropriate returned values:

`push_right(vnew)` if S is not full, sets S to be the sequence $S = \langle v_0, \dots, v_k, v_{\text{new}} \rangle$

`push_left(vnew)` if S is not full, sets S to be the sequence $S = \langle v_{\text{new}}, v_0, \dots, v_k \rangle$

`pop_right()` if S is not empty, sets S to be the sequence $S = \langle v_0, \dots, v_{k-1} \rangle$

`pop_left()` if S is not empty, sets S to be the sequence $S = \langle v_1, \dots, v_k \rangle$

[1037] For example, starting with an empty deque state, $S = \langle \rangle$, the following sequence of operations and corresponding transitions can occur. A

`push_right(1)` changes the deque state to $S = \langle 1 \rangle$. A `push_left(2)` subsequently changes the deque state to $S = \langle 2, 1 \rangle$. A subsequent `push_right(3)` changes the deque state to $S = \langle 2, 1, 3 \rangle$. Finally, a subsequent `pop_right()` changes the deque state to $S = \langle 2, 1 \rangle$.

An Array-Based Implementation

[1038] The description that follows presents an exemplary non-blocking implementation of a deque based on an underlying contiguous array data structure wherein access operations (illustratively, `push_left`, `pop_left`, `push_right` and `pop_right`) employ DCAS operations to facilitate concurrent access.

Exemplary code and illustrative drawings will provide persons of ordinary skill in the art with detailed understanding of one particular realization of the present invention; however, as will be apparent from the description herein and the breadth of the claims that follow, the invention is not limited thereto. Exemplary right-hand-side code is described in substantial detail with the understanding that left-hand-side operations are symmetric. Use herein of directional signals (e.g., left and right) will be understood by persons of ordinary skill in the art to be somewhat arbitrary. Accordingly, many other notational conventions, such as top and bottom, first-end and second-end, etc., and implementations denominated therein are also suitable.

[1039] With the foregoing in mind, an exemplary non-blocking implementation of a deque based on an underlying contiguous array data structure is illustrated with reference to **FIGS. 1A** and **1B**. In general, an array-based deque implementation includes a contiguous array $S[0 \dots \text{length_S} - 1]$ of storage locations indexed by two counters, R and L . The array, as well as the counters (or alternatively, pointers or indices), are typically stored in memory. Typically, the array S and indices R and L are stored in a same memory, although more generally, all that is required is that a particular DCAS implementation span the particular storage locations of the array and an index.

[1040] In operations on S , we assume that mod is the modulus operation over the integers (e.g., $-1 \bmod 6 = 5$, $-2 \bmod 6 = 4$, and so on). Henceforth, in the description that follows, we assume that all values of R and L are modulo length_S , which implies that the array S is viewed as being circular. The array $S[0 \dots \text{length_S} - 1]$ can be viewed as if it were laid out with indexes increasing from left to right. We assume a distinguishing value, e.g., “null” (denoted as 0 in the drawings), not occurring in the range of real data values for S . Of course, other distinguishing values are also suitable.

[1041] Operations on S proceed as follows. Initially, for empty deque state, L points immediately to the left of R . In the illustrative embodiment, indices L and R always point to the next location into which a value can be inserted. If there is a null value stored in the element of S immediately to the right of that identified by L (or respectively, in the element of S immediately to the left of that identified by R), then the deque is in the empty state. Similarly, if there is a non-null value in the element of S identified by L (respectively, in the element of S identified by R), then the deque is in the full state. **FIG. 1A** depicts an empty state and **FIG. 1B** depicts a full state. During the execution of access operations in accordance with the present invention, the use of a DCAS guarantees that on any location in the array, at most one processor can succeed in modifying the entry at that location from a “null” to a “non-null” value or vice versa.

[1042] An illustrative `pop_right` access operation in accordance with the present invention follows:

```

1      val pop_right {
2          while (true) {
3              oldR = R;
4              newR = (oldR - 1) mod length_S;
5              oldS = S[newR];
6              if (oldS == "null") {
7                  if (oldR == R)
8                      if (DCAS(&R, &S[newR],
9                          oldR, oldS, oldR, oldS))
10                     return "empty";
11              }
12              else {
13                  newS = "null";
14                  if (DCAS(&R, &S[newR],
15                      oldR, oldS, &newR, &newS))
16                     return newS;
17                  else if (newR == oldR) {
18                      if (newS == "null") return "empty";
19                  }
20              }
21          }
22      }

```

[1043] To perform a `pop_right`, a processor first reads `R` and the location in `S` corresponding to `R-1` (Lines 3-5, above). It then checks whether `S[R-1]` is null. As noted above, `S[R-1]` is shorthand for `S[R-1 mod length_S]`. If `S[R-1]` is null, then the processor reads `R` again to see if it has changed (Lines 6-7). This additional read is a performance enhancement added under the assumption that the common case is that a null value is read because another processor “stole” the item, and not because the queue is really empty. Other implementations need not employ such an enhancement. The test can be stated as follows: if `R` hasn’t changed and `S[R-1]` is null, then the deque must be empty since the location to the left of `R` always contains a value unless there are no items in the deque. However, the conclusion that the deque is empty can only be made based on an instantaneous view of `R` and `S[R-1]`. Therefore, the `pop_right` implementation employs a `DCAS` (Lines 8-10) to check if this is in fact the case. If so, `pop_right` returns an indication that the deque is empty. If not, then either the value in `S[R-1]` is no longer null or the index `R` has changed. In either case, the processor loops around and starts again, since there might now be an item to pop.

[1044] If $S[R-1]$ is not null, the processor attempts to pop that item (Lines 12-20). The `pop_right` implementation employs a DCAS to try to atomically decrement the counter R and place a null value in $S[R-1]$, while returning (via `&newR` and `&newS`) the old value in $S[R-1]$ and the old value of the counter R (Lines 13-15). Note that the overloaded variant of DCAS described above is utilized here.

[1045] A successful DCAS (and hence a successful `pop_right` operation) is depicted in FIG. 2. Initially, $S = \langle v_1, v_2, v_3, v_4 \rangle$ and L and R are as shown. Contents of R and of $S[R-1]$ are read, but the results of the reads may not be consistent if an intervening competing access has successfully completed. In the context of the deque state illustrated in FIG. 2, the competing accesses of concern are a `pop_right` or a `push_right`, although in the case of an almost empty state of the deque, a `pop_left` might also intervene. Because of the risk of a successfully completed competing access, the `pop_right` implementation employs a DCAS (lines 14-15) to check the instantaneous values of R and of $S[R-1]$ and, if unchanged, perform the atomic update of R and of $S[R-1]$ resulting in a deque state of $S = \langle v_1, v_2, v_3 \rangle$.

[1046] If the DCAS is successful (as indicated in FIG. 2), the `pop_right` returns the value v_4 from $S[R-1]$. If it fails, `pop_right` checks the reason for the failure. If the reason for the DCAS failure was that R changed, then the processor retries (by repeating the loop) since there may be items still left in the deque. If R has not changed (Line 17), then the DCAS must have failed because $S[R-1]$ changed. If it changed to null (Line 18), then the deque is empty. An empty deque may be the result of a competing `pop_left` that “steals” the last item from the `pop_right`, as illustrated in FIG. 4.

[1047] If, on the other hand, $S[R-1]$ was not null, the DCAS failure indicates that the value of $S[R-1]$ has changed, and some other processor(s) must have completed a pop and a push between the read and the DCAS operation. In this case, `pop_right` loops back and retries, since there may still be items in the deque. Note that Lines 17-18 are an optimization, and one can instead loop back if the DCAS fails. The optimization allows detection of a possible empty state without going through the

loop, which in case the queue was indeed empty, would require another DCAS operation (Lines 6-10).

[1048] To perform a `push_right`, a sequence similar to `pop_right` is performed. An illustrative `push_right` access operation in accordance with the present invention follows:

```

1      val push_right(val v) {
2          while (true) {
3              oldR = R;
4              newR = (oldR + 1) mod length_S;
5              oldS = S[oldR];
6              if (oldS != "null") {
7                  if (oldR == R)
8                      if (DCAS(&R, &S[oldR],
9                          oldR, oldS, oldR, oldS))
10                         return "full";
11              }
12              else {
13                  newS = v;
14                  if DCAS(&R, &S[oldR],
15                      oldR, oldS, &newR, &newS)
16                      return "okay";
17                  else if (newR == oldR)
18                      return "full";
19              }
20          }
21      }

```

[1049] Operation of `pop_right` is similar to that of `push_right`, but with all tests to see if a location is null replaced with tests to see if it is non-null, and with S locations corresponding to an index identified by, rather than adjacent to that identified by, the index. To perform a `push_right`, a processor first reads R and the location in S corresponding to R (Lines 3-5, above). It then checks whether S[R] is non-null. If S[R] is non-null, then the processor reads R again to see if it has changed (Lines 6-7). This additional read is a performance enhancement added under the assumption that the common case is that a non-null value is read because another processor "beat" the processor, and not because the queue is really full. Other implementations need not employ such an enhancement. The test can be stated as follows: if R hasn't changed and S[R] is non-null, then the deque must be full since the location identified by R always contains a null value unless the deque is full.

However, the conclusion that the deque is full can only be made based on an instantaneous view of R and S [R]. Therefore, the push_right implementation employs a DCAS (Lines 8-10) to check if this is in fact the case. If so, push_right returns an indication that the deque is full. If not, then either the value in S [R] is no longer non-null or the index R has changed. In either case, the processor loops around and starts again.

[1050] If S [R] is null, the processor attempts to push value, v, onto S (Lines 12-19). The push_right implementation employs a DCAS to try to atomically increment the counter R and place the value, v, in S [R], while returning (via &newR) the old value of index R (Lines 14-16). Note that the overloaded variant of DCAS described above is utilized here.

[1051] A successful DCAS and hence a successful push_right operation into an empty deque is depicted in FIG. 3. Initially, $S = \langle \rangle$ and L and R are as shown. Contents of R and of S [R] are read, but the results of the reads may not be consistent if an intervening competing access has successfully completed. In the context of the empty deque state illustrated in FIG. 3, the competing access of concern is another push_right, although in the case of non-empty state of the deque, a pop_right might also intervene. Because of the risk of a successfully completed competing access, the push_right implementation employs a DCAS (lines 14-15) to check the instantaneous values of R and of S [R] and, if unchanged, perform the atomic update of R and of S [R] resulting in a deque state of $S = \langle v_1 \rangle$. A successful push_right operation into an almost-full deque is illustrated in the transition from deque states of FIGS. 5B and 5C.

[1052] In the final stage of the push_right code, in case the DCAS failed, there is a check using the value returned (via &newR) to see if the R index has changed. If it has not, then the failure must be due to a non-null value in the corresponding element of S, which means that the deque is full.

[1053] Pop_left and push_left sequences correspond to their above described right hand variants. An illustrative pop_left access operation in accordance with the present invention follows:


```

1      val pop_left {
2          while (true) {
3              oldL = L;
4              newL = (oldL + 1) mod length_S;
5              oldS = S[newL];
6              if (oldS == "null") {
7                  if (oldL == L)
8                      if (DCAS(&L, &S[newL],
9                          oldL, oldS, oldL, oldS))
10                     return "empty";
11              }
12              else {
13                  newS = "null";
14                  if (DCAS(&L, &S[newL],
15                      oldL, oldS, &newL, &newS))
16                      return newS;
17                  else if (newL == oldL) {
18                      if (newS == "null") return "empty";
19                  }
20              }
21          }
22      }

```

[1054] An illustrative push_left access operation in accordance with the present invention follows:

```

1      val push_left(val v) {
2          while (true) {
3              oldL = L;
4              newL = (oldL - 1) mod length_S;
5              oldS = S[oldL];
6              if (oldS != "null") {
7                  if (oldL == L)
8                      if (DCAS(&L, &S[oldL],
9                          oldL, oldS, oldL, oldS))
10                     return "full" ;
11              }
12              else {
13                  newS = v;
14                  if (DCAS(&L, &S[oldL],
15                      oldL, oldS, &newL, &newS))
16                      return "okay";
17                  else if (newL == oldL)
18                      return "full";
19              }
20          }
21      }

```

[1055] **FIGS. 5A, 5B and 5C** illustrate operations on a nearly full deque including a `push_left` operation (**FIG. 5B**) and a `push_right` operation that result in a full state of the deque (**FIG. 5C**). Notice that L has wrapped around and is "to-the-right" of R, until the deque becomes full, in which case again L and R cross. This switching of the relative location of the L and R pointers is somewhat confusing and represents a limitation of the linear presentation in the drawings. However, in any case, it should be noted that each of the above described access operations (`push_left`, `pop_left`, `push_right` and `pop_right`) can determine the state of the deque, without regard to the relative locations of L and R, but rather by examining the relation of a given index (R or L) to the value in a corresponding element of S.

A Linked-List-Based Implementation

[1056] The previous description presents an array-based deque implementation appropriate for computing environments in which, or for which, the maximum size of the deque can be predicted in advance. In contrast, the linked-list-based implementation described below avoids fixed allocations and size limits by allowing dynamic allocation of storage for elements of a represented sequence.

[1057] Although a variety of linked-list-based concurrent shared object implementations are envisioned, a non-blocking implementation of a deque based on an underlying doubly-linked list is illustrative. In one such implementation, access operations (illustratively, `push_left`, `pop_left`, `push_right` and `pop_right`) as well as auxiliary delete operations (`delete_left` and `delete_right`) employ DCAS operations to facilitate non-blocking concurrent access to the deque. Exemplary code and illustrative drawings will provide persons of ordinary skill in the art with a detailed understanding of one particular realization of the present invention; however, as will be apparent from the description herein and the breadth of the claims that follow, the invention is not limited thereto.

[1058] Aspects of the deque implementation described herein will be understood by persons of ordinary skill in the art to provide a superset of structures and techniques which may also be employed in less complex concurrent shared object implementations, such as LIFO-stacks, FIFO-queues and concurrent shared objects

(including dequeues) with simplified access semantics. Furthermore, although the description that follows emphasizes doubly-linked list implementations, persons of ordinary skill in the art will recognize that the techniques described may also be exploited in simplified form for concurrent shared objects based on a singly-linked list.

[1059] With the forgoing in mind, and without limitation, the description that follows focuses on an exemplary linearizable, non-blocking concurrent deque implementation based on an underlying doubly-linked list of nodes. Each node includes two link pointers and a value field as follows:

```
typedef node {
    pointer *L;
    pointer *R;
    val_or_null_or_SentL_or_SentR value;
}
```

[1060] It is assumed that there are three distinguishing values (called null, sentL, and sentR) that can be stored in the value field of a node, but which are never pushed onto the deque.

[1061] In an exemplary doubly-linked list implementation, two distinguishing nodes, called "sentinels," are employed. The left sentinel is at a known fixed address SL. The left sentinel's L pointer is not used and its value field contains the distinguishing value, sentL. Similarly, the right sentinel is at a known fixed address SR. The right sentinel's R pointer is also not used and its value field contains the distinguishing value, sentR. Although the sentinel node technique of identifying list ends is presently preferred, other techniques consistent with the concurrency control described herein may also be employed.

[1062] In general, a node can be removed from the list in response to invocation of a pop_right or pop_left operation in two separate, atomic steps. First, the node is "logically" deleted, e.g., by replacing its value with "null" and setting a deleted indication to signify the presence of a logically deleted node. Second, the node is "physically" deleted by modifying pointers so that the node is no longer in the doubly-linked chain of nodes and by resetting the deleted indication. In each case, a

synchronization primitive, preferably a DCAS, can be employed to ensure proper synchronization with competing push, pop, and delete operations.

[1063] If a process that is removing a node is suspended between completion of the logical deletion step and the physical deletion step, then any other process can perform the physical deletion step or otherwise work around the fact that the second step has not yet been performed. In some realizations of a deque, the physical deletion is performed as part of a next same end push or pop operation. In other realizations, physical deletion may be performed as part of the initiating pop operation.

[1064] In one deque realization, deleted indications are stored in the sentinel node corresponding to the end of the list from which a node has been logically removed. One presently preferred representation of the deleted indication is as a deleted bit encoded as part of a sentinel node's pointer to the body of the linked list. For example,

```
typedef pointer {
    node *ptr;
    boolean deleted;
}
```

[1065] Assuming sufficient pointer alignment to free a low-order bit, the pointer structure may be represented as a single word, thereby facilitating atomic update of the sentinel node's pointer to the list body, the deleted bit, and a node value, all using a double-word compare and swap (DCAS) operation. Nonetheless, other encodings are also suitable. For example, the deleted indication may be separately encoded at the cost, in some implementations, of more complex synchronization (e.g., N-word compare-and-swap operations) or by introducing a special dummy type "delete-bit" node, distinguishable from the regular nodes described above. In one such configuration, illustrated in FIG. 6, each processor has a dummy node for the left and one for the right. Given such dummy nodes, an indirect reference to a list body node via a dummy node can be used to encode a true value of the deleted indication, whereas a direct reference can represent a false value. Particular deleted indications are implementation specific and any of a variety of encodings are suitable.

However, for the sake of illustration and without loss of generality, a deleted bit encoding is assumed for the description that follows.

[1066] Operations on a linked-list encoded deque proceed as follows. An initial empty state of the deque is typically represented as illustrated in FIG. 7A, i.e., with $SR \rightarrow L == SL$ and $SL \rightarrow R == SR$. However, as will become apparent from the description that follows, several other states of the linked list correspond to an empty deque, albeit represented as a list with one or two logically, but not yet physically, deleted nodes. FIGS. 7B, 7C and 7D illustrate these additional empty states with deleted bits encoded as part of corresponding sentinel node's pointers to a null value element of the linked list.

[1067] Push and pop operations are now described, each in turn. Both push and pop operations use an auxiliary delete operation, which is described last. Exemplary right hand code (e.g., `pop_right`, `push_right`, and `delete_right`) is described in substantial detail with the understanding that left-hand-side operations (e.g., `pop_left`, `push_left`, and `delete_left`) are symmetric. As before, use of directional signals (e.g., left and right) will be understood by persons of ordinary skill in the art to be somewhat arbitrary. Accordingly, many other notational conventions, such as top and bottom, first-end and second-end, etc., and implementations denominated therein are also suitable.

[1068] An illustrative `pop_right` access operation in accordance with the present invention follows:

```

1      val pop_right() {
2          while (true) {
3              oldL = SR->L;
4              v = oldL.ptr->value;
5              if (v == "SentL") return "empty";
6              if (oldL.deleted == true)
7                  delete_right();
8              else if (v == "null") {
9                  if (DCAS(&SR->L, &oldL.ptr->value,
10                      oldL, v, oldL, v))
11                      return "empty";
12              }
13              else {
14                  newL.ptr = oldL.ptr;

```

```

15         newL.deleted = true;
16         if (DCAS(&SR->L, &oldL.ptr->value,
17                 oldL, v, newL, "null"))
18             return v;
19     }
20 }
21

```

[1069] To perform a `pop_right`, an executing processor first reads `SR->L` and the value (`oldL.ptr->value`) of the node identified thereby (lines 3-4, above). The processor then checks the identified node for a `SentL` distinguishing value (line 5). If present, the deque has the empty state illustrated in **FIG. 7A** and `pop_right` returns. If not, the processor checks whether the `deleted` bit of the right sentinel's `L` pointer is true. If so, then the processor invokes the `delete_right` operation to remove the null node on the right-hand side, and then retries the pop. If the `deleted` bit of the right sentinel's `L` pointer is false, then the processor checks whether the node to be popped encodes a "null" value (Line 8). If so, the deque could have the empty state illustrated in **FIG. 7C** or the initially read `SR->L` and `v` may not represent a valid instantaneous state. To test for the empty state, `pop_right` performs an atomic check, using a `DCAS` operation, for presence of both a "null" value in the node and a false `deleted` bit encoded in the pointer to that node from the right sentinel (Lines 9-11). If the `DCAS` is successful, the deque is in the empty state illustrated in **FIG. 7C** (i.e., a `pop_left` execution has successfully completed, but `delete_left` has not) and `pop_right` returns. Otherwise, the deque must have been modified between the original reads and the `DCAS` test, in which case `pop_right` loops and retries.

[1070] Finally, there is the case in which the `deleted` bit is false and `v` is not null, as in the deque state illustrated in **FIG. 8B**. Using a `DCAS`, `pop_right` atomically swaps `v` out from the node, changing its value to "null," while at the same time changing the `deleted` bit in the node identifying pointer of the right sentinel (`SR->L`) to true (Lines 14-17). If the `DCAS` fails, then either the left pointer of the right sentinel (`SR->L`) no longer points to the node for which a pop was attempted (such as if a competing concurrent `push_right` successfully completed between one of the original reads and the `DCAS` test) or the value of the identified node has been set to

“null” (e.g., by successful completion of a competing concurrent `pop_right` or `pop_left`). In either case, `pop_right` loops back to retry. However, if the DCAS is successful (Line 18), `pop_right` returns `v` as the result of the pop, leaving the deque in a state, such as illustrated in **FIG. 8D**, wherein the right sentinel’s deleted bit is true, indicating that the node has been logically deleted. Typically, the next `pop_right` or `push_right` will call the `delete_right` operation to perform the physical deletion. However, in some implementations, `pop_right` may invoke `delete_right` before returning.

[1071] An illustrative `push_right` access operation in accordance with the present invention follows:

```

1      val push_right(val v) {
2          newL.ptr = new Node();
3          if (newL.ptr == "null") return "full";
4          newL.deleted = false;
5          while (true) {
6              oldL = SR->L;
7              if (oldL.deleted == true)
8                  delete_right();
9              else {
10                 newL.ptr->R.ptr = SR;
11                 newL.ptr->R.deleted = false;
12                 newL.ptr->L = oldL;
13                 newL->value = v;
14                 oldLR.ptr = SR;
15                 oldLR.deleted = false;
16                 if (DCAS(&SR->L, &SR->L.ptr->R,
17                         oldL, oldLR, newL, newL))
18                     return "okay";
19             }
20         }
21     }

```

[1072] Execution of the `push_right` operation is now described with reference to **FIGS. 9A** and **9B** and the above exemplary code. `Push_right` begins by obtaining and initializing a new node (lines 2-4). The operation then reads `SR->L` and checks if the deleted bit encoded in the right sentinel is true (lines 6-7). If so, `push_right` invokes `delete_right` to physically delete the null node to which the right sentinel’s left pointer (`SR->L`) points and retries. If instead, the deleted bit is false, `push_right` initializes value and left and right pointers of the new

node to splice the new node into the list between the right sentinel and its left neighbor (lines 10-13). Using a DCAS, `push_right` atomically updates the right sentinel's left pointer (`SR->L`) and the left neighbor's right pointer (`SR->L.ptr->R`). If the DCAS is successful, the splice is completed as illustrated in **FIG. 9B**. Otherwise, deque state has changed since `SR->L` was read in a way that affects the consistency of the pointers (e.g., due to successful completion of a competing concurrent `push_right`, `pop_right` or `pop_left`) in which case `push_right` loops back and retries.

[1073] An illustrative `delete_right` operation in accordance with the present invention follows:

```

1      delete_right() {
2          while (true) {
3              oldL = SR->L;
4              if (oldL.deleted == false) return;
5              oldLL = oldL.ptr->L.ptr;
6              if (oldLL->value != "null") {
7                  oldLLR = oldLL->R;
8                  if (oldL.ptr == oldLLR.ptr) {
9                      newR.ptr = SR;
10                     newR.deleted = false;
11                     if (DCAS(&SR->L, &oldLL->R,
12                             oldL, oldLLR, oldLL, newR))
13                         return;
14                 }
15             }
16             else { /* there are two null items */
17                 oldR = SL->R;
18                 newL.ptr = SL;
19                 newL.deleted = false;
20                 newR.ptr = SR;
21                 newR.deleted = false;
22                 if (oldR.deleted)
23                     if (DCAS(&SR->L, &SL->R,
24                             oldL, oldR, newL, newR))
25                         return;
26             }
27         }
28     }

```

[1074] Execution of the `delete_right` operation is now described with reference to **FIGS. 8A** and **8C** and the above exemplary code. `Delete_right` begins by

checking that the left pointer in the right sentinel has its deleted bit set to true (line 4). Otherwise, `delete_right` returns.

[1075] If the deleted bit is true, the next step is to determine the state of the deque. In general, the deque state may be empty as illustrated in **FIGS. 7B** or **7D** or may include one or more non-null elements (e.g., as illustrated in **FIG. 8A**). To determine which, `delete_right` obtains a pointer (`oldLL`) to the node immediately left of the node to be deleted. `Delete_right` then checks the value in the node identified by the pointer `oldLL` (Line 6). In general, this node may (1) have a non-null value, (2) be the left sentinel, or (3) have a null value. In the first two cases, which correspond respectively to the states depicted in **FIGS. 8A** and **7B**, the previously read right sentinel pointer (`oldL.ptr`) is compared against the right pointer of the node identified by `oldLL` (i.e., `oldLLR.ptr`). If the pointers are unequal, the deque has been modified such that `delete_right` pointer values are inconsistent and should be read again. Accordingly, `delete_right` loops and retries. If however, the pointers are equal, `delete_right` employs a DCAS to atomically swap pointers so that `SR` and `oldLL` point to each other, excising the null node from the list. **FIG. 8C** illustrates successful completion of a `delete_right` operation on the initial deque state illustrated in **FIG. 8A**.

[1076] The case of the null value is a bit different. A null value indicates that deque state is empty with two null elements as illustrated in **FIG. 7D**. To delete both null elements, `delete_right` checks `oldR.deleted`, the deleted bit encoded in the right pointer of the left sentinel, to see if the deleted bits in both sentinels are true (line 22). If so, `delete_right` attempts to point the sentinels to each other using a DCAS (lines 23-24). In case of failure, `delete_right` loops and retries until the deletion is completed.

[1077] The most interesting case occurs when there are two null nodes and a `delete_left` about to be executed from the left, concurrent with a `delete_right` about to be executed from the right. A variety of scenarios may develop depending on the order of operations. However, the scenario depicted in **FIG. 10** is illustrative. In general, the deque states illustrated in **FIG. 10** can occur if

a `delete_left` (which is symmetric with `delete_right`) starts first, e.g., reading the value of the node immediately right of the node it is to delete (`oldRR->value`) while that value is still non-null, but just before a concurrent execution of `pop_right` sets the value to null. The `delete_left` (symmetrically as described above with reference to `pop_right`) attempts to delete a single null node using a DCAS to atomically update the left sentinel's right pointer and the right-most null node's left pointer. (Note that `delete_left` is unaware that the right-most of the two null nodes has been popped and is in fact contains a null value.) Concurrently, the `delete_right`, which started later following the `pop_right`, detects the two empty nodes and attempts to delete both null nodes using a DCAS to atomically update the pointers of the left and right sentinels to point to the other. As illustrated in **FIG. 10**, the DCAS operations overlap on the pointer in the left sentinel and two outcomes are possible.

[1078] If `delete_left` executes its DCAS first, `delete_left`'s attempted single node delete succeeds and `delete_right`'s attempted double node delete fails. The `deleted` bit of the right sentinel remains true and a single null node remains for deletion by `delete_right` on its next pass. If instead, `delete_right` executes its DCAS first, `delete_right`'s attempted double node delete succeeds, resulting in a deque state as illustrated in **FIG. 7A**. `Delete_left`'s attempted single node delete fails. The `deleted` bits of both right and left sentinels are set to false and `delete_left` returns on its next pass based on the false state of the left sentinel's `deleted` bit.

[1079] Based on the above description of illustrative right-hand variants of push, pop and delete operations, persons of ordinary skill in the art will immediately appreciate operation of the left-hand variants. Indeed, `Pop_left`, `push_left` and `delete_left` sequences are symmetric to their above described right hand variants. An illustrative `pop_left` access operation in accordance with the present invention follows:

```

1      val pop_left() {
2          while (true) {
3              oldR = SL->R;
```

```

4      v = oldR.ptr->value;
5      if (v == "SentR") return "empty";
6      if (oldR.deleted == true)
7          delete_left();
8      else if (v == "null") {
9          if (DCAS(&SL->R, &oldR.ptr->value,
10               oldR, v, oldR, v))
11              return "empty";
12      }
13      else {
14          newR.ptr = oldR.ptr;
15          newR.deleted = true;
16          if (DCAS(&SL->R, &oldR.ptr->value,
17               oldR, v, newR, "null"))
18              return v;
19      }
20  }
21  }

```

[1080] An illustrative push_left access operation in accordance with the present invention follows:

```

1      val push_left(val v) {
2          newR.ptr = new Node();
3          if (newR.ptr == "null") return "full";
4          newR.deleted = false;
5          while (true) {
6              oldR = SL->R;
7              if (oldR.deleted == true)
8                  delete_left();
9              else {
10                 newR.ptr->L.ptr = SL;
11                 newR.ptr->L.deleted = false;
12                 newR.ptr->R = oldR;
13                 newR->value = v;
14                 oldRL.ptr = SL;
15                 oldRL.deleted = false;
16                 if (DCAS(&SL->R, &SL->R.ptr->L,
17                     oldR, oldRL, newR, newR))
18                     return "okay";
19             }
20         }
21     }

```

[1081] An illustrative delete_left operation in accordance with the present invention follows:

```

1      delete_left() {

```

```

2      while (true) {
3          oldR = SL->R;
4          if (oldR.deleted == false) return;
5          oldRR = oldR.ptr->R.ptr;
6          if (oldRR->value != "null") {
7              oldRRL = oldRR->L;
8              if (oldR.ptr == oldRRL.ptr) {
9                  newL.ptr = SL;
10                 newL.deleted = false;
11                 if (DCAS(&SL->R, &oldRR->L,
12                         oldR, oldRRL, oldRR, newL))
13                     return;
14             }
15         }
16         else { /* there are two null items */
17             oldL = SR->L;
18             newR.ptr = SR;
19             newR.deleted = false;
20             newL.ptr = SL;
21             newL.deleted = false;
22             if (oldL.deleted)
23                 if (DCAS(&SR->L, &SL->R,
24                         oldL, oldR, newL, newR))
25                     return;
26         }
27     }
28 }

```

[1082] While the invention has been described with reference to various embodiments, it will be understood that these embodiments are illustrative and that the scope of the invention is not limited to them. Many variations, modifications, additions, and improvements are possible. Plural instances may be provided for components described herein as a single instance. Finally, boundaries between various components, operations and data stores are somewhat arbitrary, and particular operations are illustrated in the context of specific illustrative configurations. Other allocations of functionality are envisioned and may fall within the scope of claims that follow. Structures and functionality presented as discrete components in the exemplary configurations may be implemented as a combined structure or component. These and other variations, modifications, additions, and improvements may fall within the scope of the invention as defined in the claims that follow.